Is there something out there?

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Bayesian analysis is one principle and one measure of uncertainty (probability) which can be used for

- Probabilistic reasoning
- Learning from data
- Quantifying uncertainty
- Making decisions
- Integrating expert knowledge with data

Possible interpretations of probability that works in a Bayesian Analysis

- Personal (subjective) probability
- Logical probability

*The "probability on relative frequency interpretation" states that randomness or inherent variability can be expressed by relative frequencies, which can be uncertain and this uncertainty is quantified by probability.

Is there something there?

You and me have different judgments if there is something there
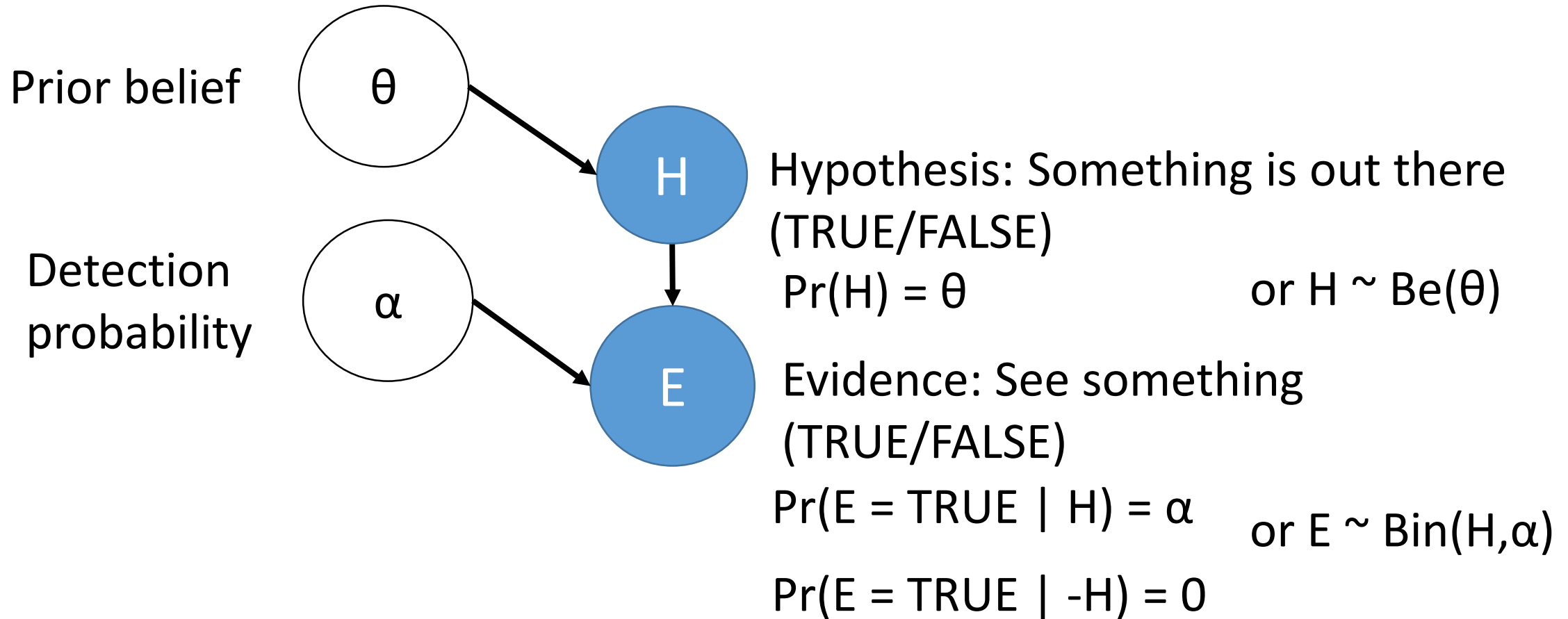You look over the shoulder to see if something is there
You did not see anything?
Are you sure there isn't something there?
Keep walking slowly or run !!!

The decision to walk slowly or to run depends on my

- Fear of that someting and how much I don't like to run compared to walk slowly

- Prior belief in that there is something there - θ

- Trust in data - α

Prior belief θ

Detection probability α

Hypothesis: Something is out there (TRUE/FALSE)

$Pr(H) = \theta$ or H ~ Be(θ)

Evidence: See something (TRUE/FALSE)

$Pr(E = TRUE \mid H) = \alpha$ or E ~ Bin(H,α)

$Pr(E = TRUE \mid -H) = 0$

$$P(H|E = 0) = \frac{(1 - \alpha)\theta}{1 - \alpha\theta}$$

Two examples of Bayesian analysis

Parameters "with no uncertainty"
- θ is scalar
- Bayesian Network
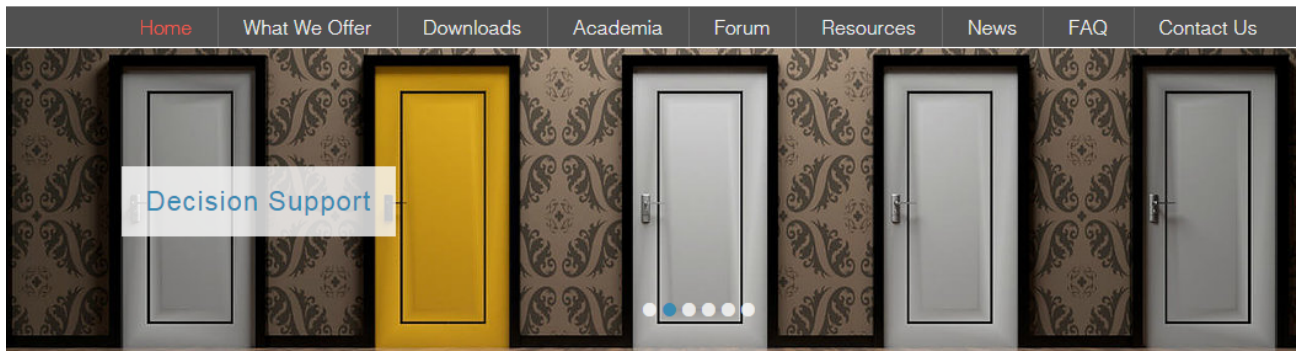
Parameters "with uncertainty"
- θ is specified with uncertainty
- Bayesian model updated with MCMC sampling
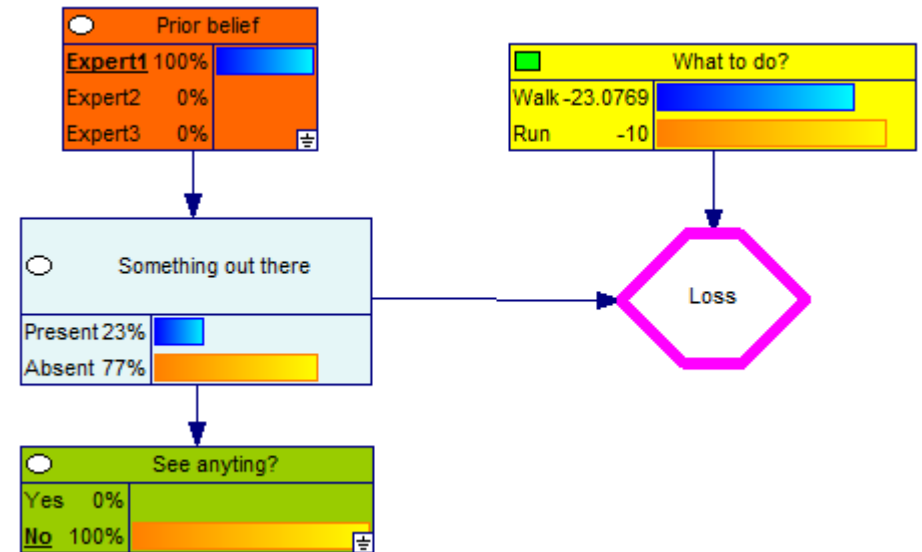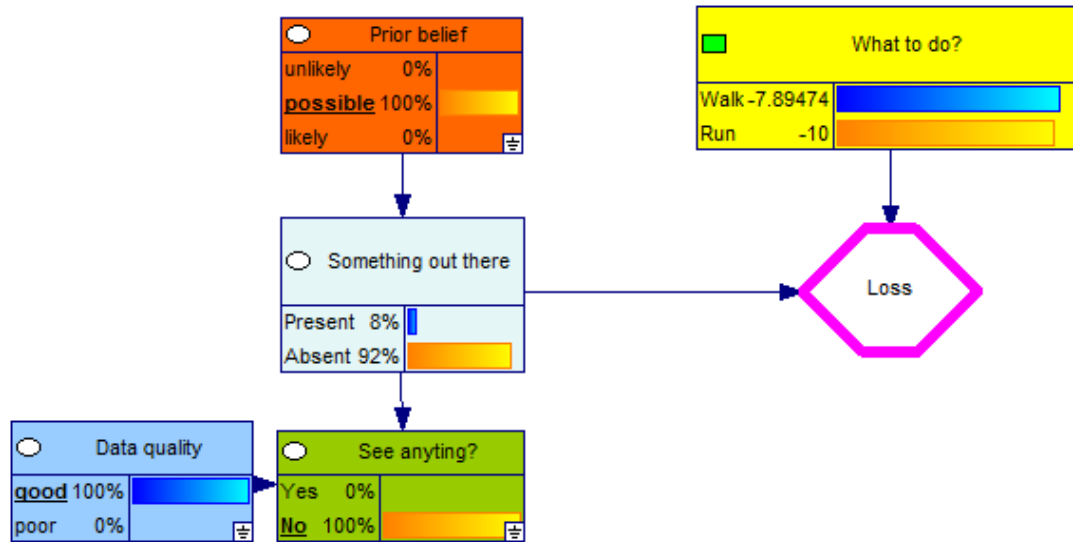
Parameters "with no uncertainty"

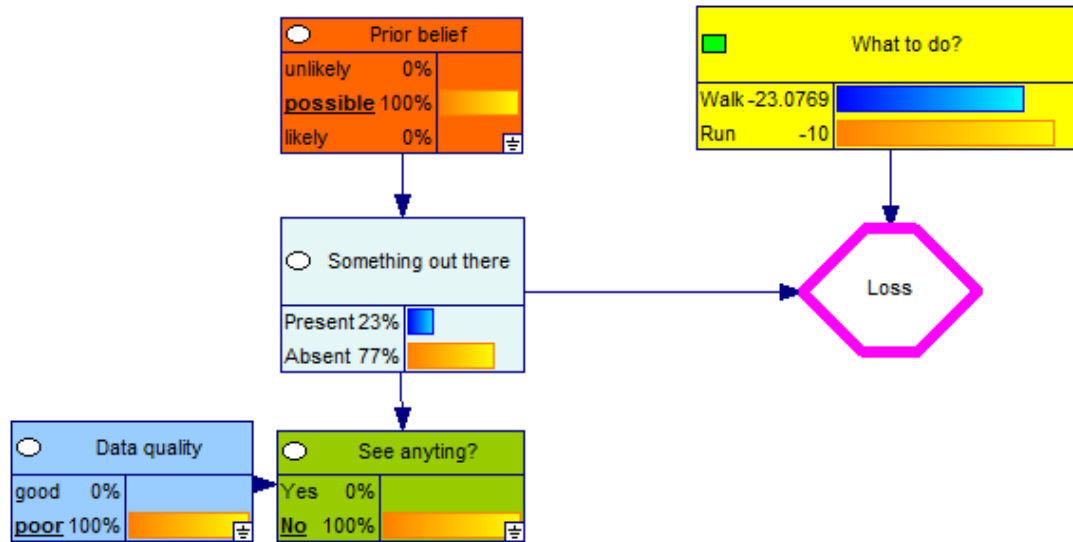- GeNie
- https://www.bayesfusion.com/

When to run?

According to the Bayesian Decision Analysis I should keep on walking slowly when I trust my observation is good and Good data quality and I believe it is possible that there is something there

When to run?

If I dont trust my observation the Bayes optimal decison is to run.

This is what I usually do

*Considering situations when I walk and run can help me to acutally estimate my belief that there is something there (this is what is usually behind the betting interpreation of probability)
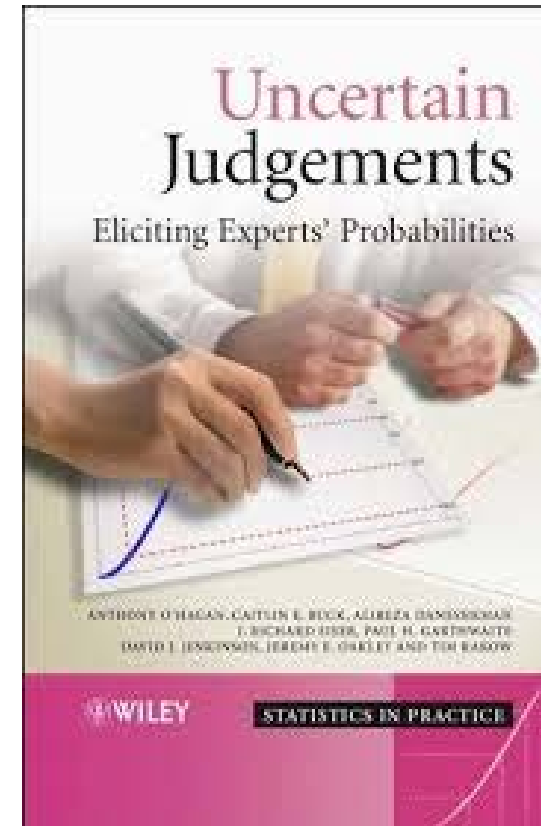
Parameters "with uncertainty"

- Is my belief θ really equal to 0.3 or is it 0.29.....

- How to specify my θ?

- Use Expert Knowledge Elicitation

Direct methods of Expert Knowledge Elicitation

- MATCH Uncertainty Elicitation
  Tool: optics.eee.nottingham.ac.uk/match/uncertainty.php#

- SHELF R-package

Indirect methods for EKE
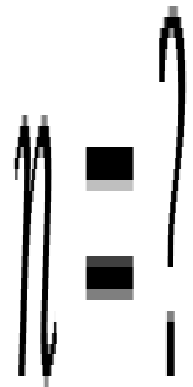
- Equivalent Prior Sample (EPS)
  - *What is the expected frequency of the event?*
  - *What is the size a sample that you imagine to have behind this estimate?*
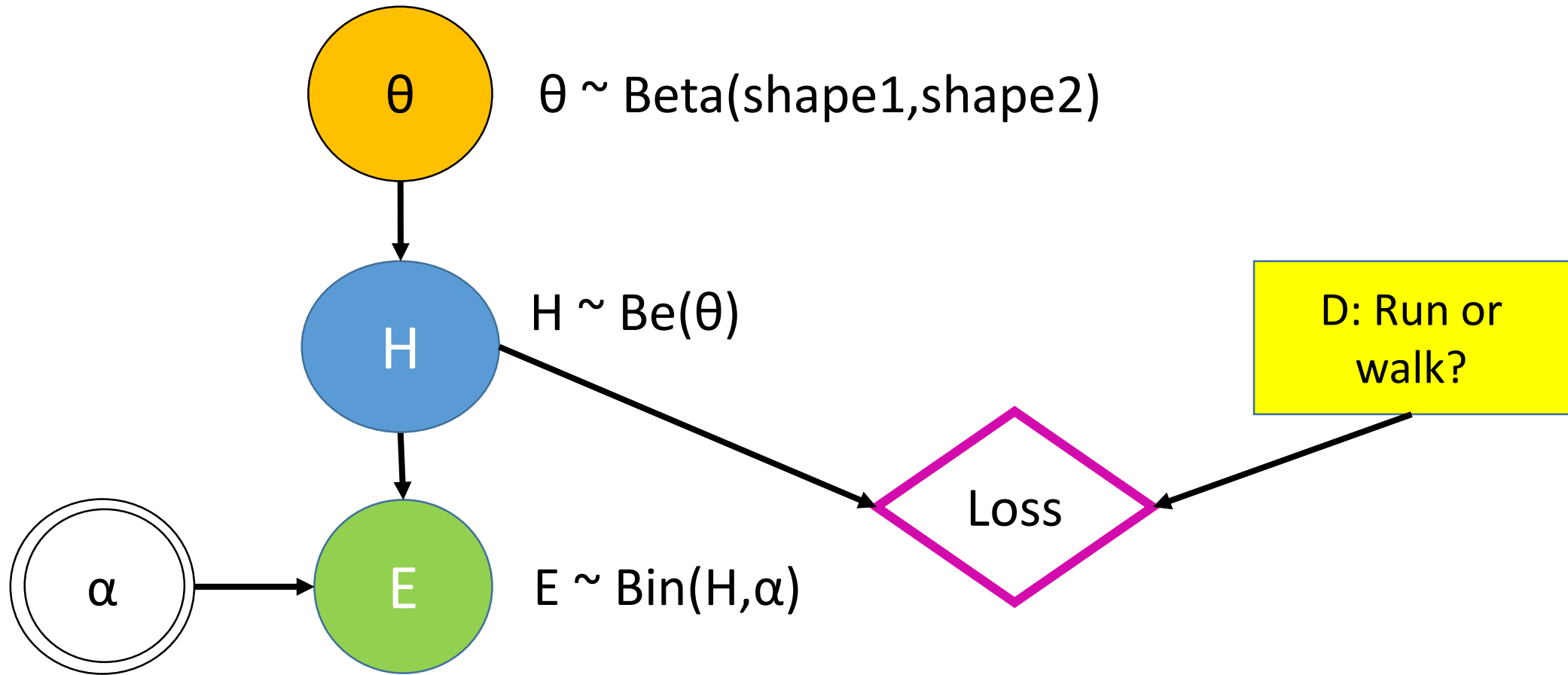
$$\frac{x}{n} = ? \qquad n = ?$$

- Hypothetical Future Sample (HFS)
  - *In a future sample of size 100 – in how many times has the event occured?*

$$n = 100 \qquad x = ?$$

The graphical model

$\theta \sim$ Beta(shape1,shape2)

$H \sim Be(\theta)$

$E \sim Bin(H,\alpha)$

Loss

D: Run or walk?

Bayesian model in JAGs

```
model {

  theta ~ dbeta(shape1, shape2)


  H ~ dbern(theta)


  E ~ dbinom(alpha,H)

  Loss_walk = -100*H


}"
```

Specifying the prior
x/n = 0.3
n = 100
shape1 = n · x/n
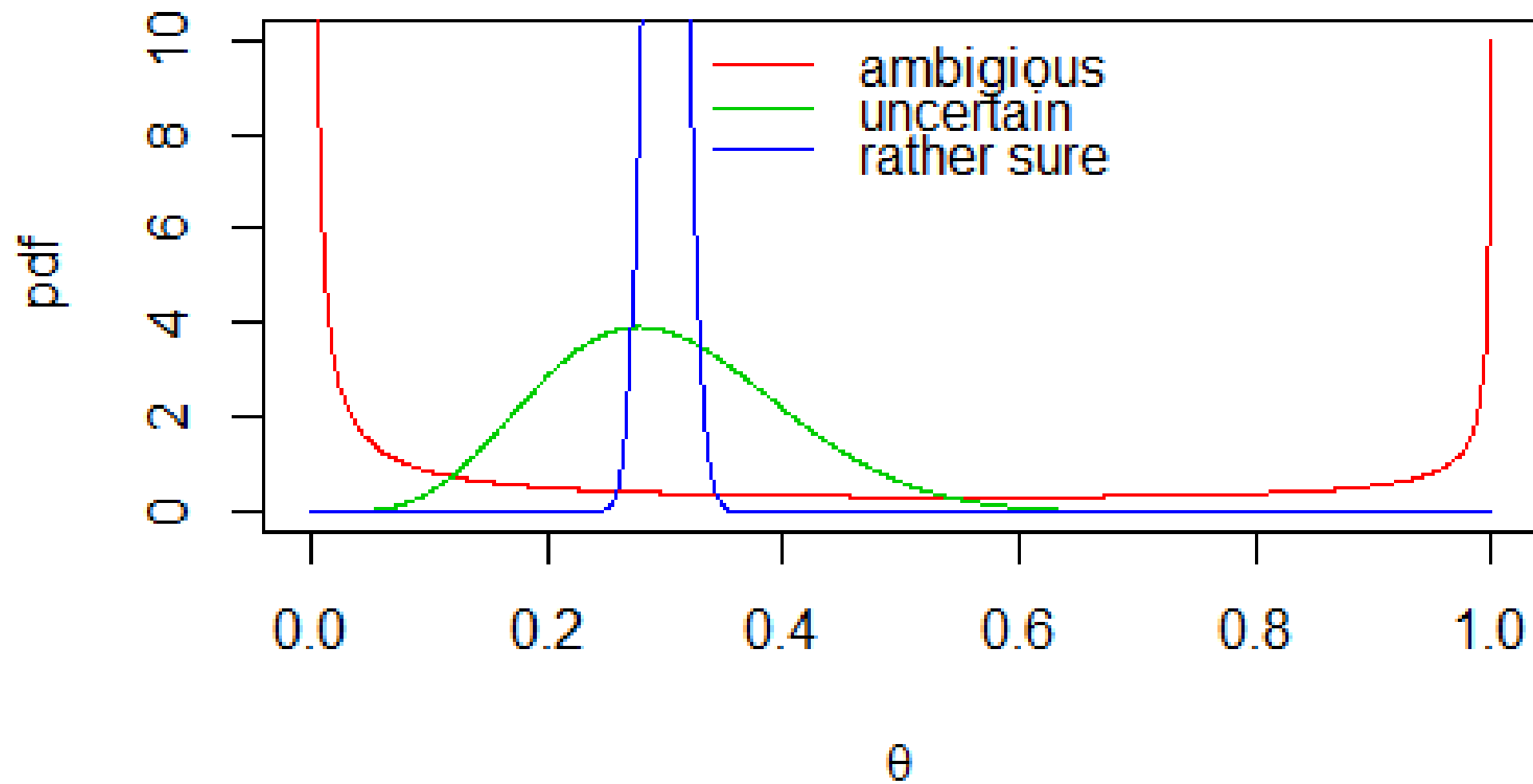shape2 = n · (1-x/n)

Loss_run = -10

priors

Legend:
- ambigious (red)
- uncertain (green)
- rather sure (blue)

x-axis: θ

y-axis: pdf

So

No uncertainty in parameter

- The single observation has an influence when
  - my belief is not too low or too high
  - the quality in data high

Uncertainty in parameter

- Adding uncertainty changes changes the expected loss depending on the distribution for uncertainty
- The single observation has an influence when
  - the prior mean is not too strong to one side
  - the belief of the hypothesis is very weak
  - the quality in data high