Modeling the growth of Swedish Scots pines

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Motivation

Swedish Scots pine west of Bottnaryd, Sweden

- Forestry, environmental and conservation planning
  - Silvicultural options: when and where to plant or cut down trees?
  - Predictions on future yields
- Enumeration and mapping of trees in large forests costly
  - Collect data from e.g. circular sample plots
  - Model forest dynamics

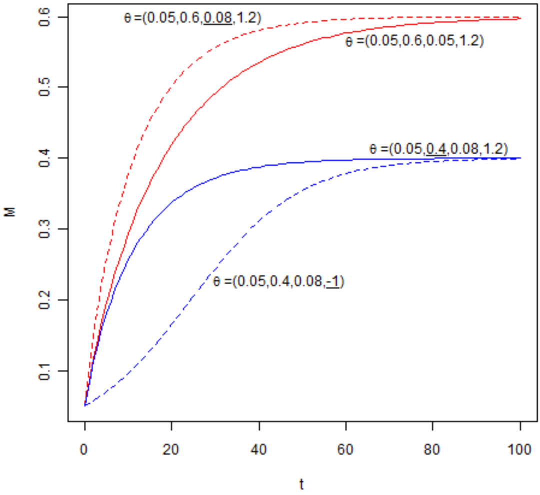
Modeling the open growth of trees

Time dependent size M_i at random location X_i on the i^{th} plot, $i \in \{1, \ldots, 2933\}$, where with Richards growth function (RGF)

$$M_i = K \left(1 + \left(\left(\frac{M(0)}{K} \right)^{\delta} - 1 \right) e^{-\lambda t_i} \right)^{\frac{1}{\delta}}$$

- carrying capacity $K > 0$
- initial size $M(0) = 0.05$
- intrinsic (open) growth rate $\lambda > 0$
- $\delta \neq 0, 1$ shape parameter

RGF: Parameters $\theta = (M(0), K, \lambda, \delta)$

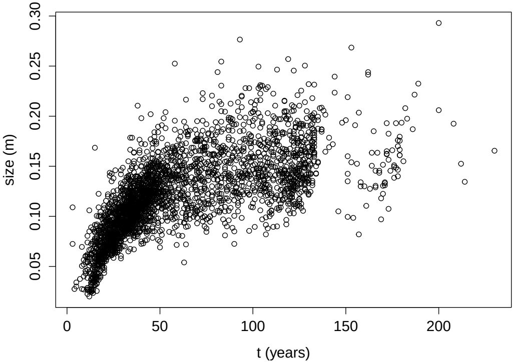


Open growth data example

plot	SI	t	m(t)
1406	20	22	0.0765
2881	18	26	0.0885
1255	18	59	0.1495
1894	20	24	0.0830
879	18	98	0.1220
1631	21	26	0.0855

Example open growth data including location (plot), site index (SI), age of largest tree (t) and its size measured in radius at breast height (m(t))

Open growth data



Open growth data from 2933 plots

Model assumptions

For observations of $M_i(t)$ we assume for $K = \alpha_0 + \alpha_1 \cdot SI$

$$m_i = K\left(1 + \left(\left(\frac{0.05}{K}\right)^{\delta} - 1\right)e^{-\lambda t_i}\right)^{\frac{1}{\delta}} + \epsilon_i = f(t_i, \theta) + \epsilon_i$$

with $\epsilon_i \sim N(0, \sigma^2)$ i.i.d. random error such that

$$m_i \sim N(f(t_i, \theta), \sigma^2) \text{ i.i.d.}$$

with $\theta = (\alpha_0, \alpha_1, \lambda, \delta)$ and constant variance σ^2

Three modeling approaches

Model O: Frequentist with only fixed effects

$$l(m|\theta,\sigma) = \prod_{i=1}^{n} p(m_i|\theta,\sigma^2)$$

Model F: Frequentist with mixed effects

$$m_i = K\left(1 + u_i + \left(\left(\frac{0.05}{K}\right)^{\delta} - 1\right)e^{-\lambda t_i}\right)^{\frac{1}{\delta}} + \epsilon_i$$

$u_i \sim N(0,\sigma_u^2)$ i.i.d.; independent of ϵ:

$$l(m|\theta,\sigma) = \prod_{i=1}^{n} \int p(m_i|\theta,\sigma^2,u_i)q(u_i|\sigma_u^2)du_i$$

Three modeling approaches ctd.

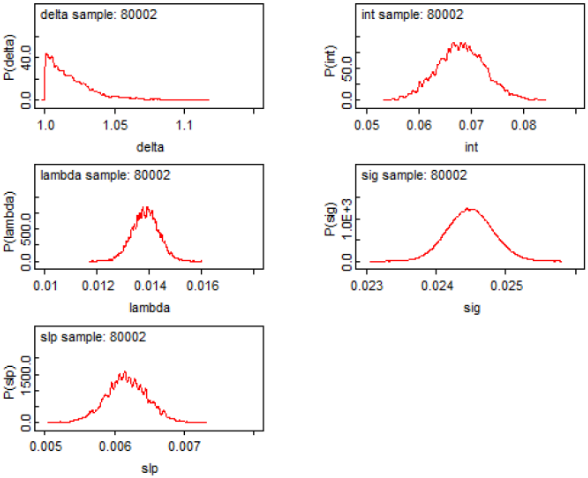
Model B: Two-level hierarchical Bayes model

$$p(\theta, \sigma | m) \propto l(m | \theta, \sigma) q(\theta, \sigma)$$

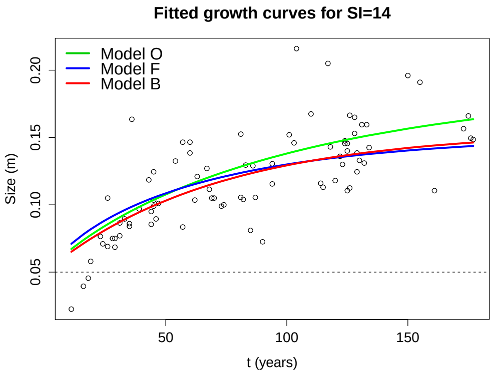
with priors

$$\alpha_0, \alpha_1 \sim U(0,1),\ \lambda \sim U(0.00001, 0.1),\ \delta \sim U(1,3),$$
$$\sigma \sim \Gamma^{-1}(0.1, 0.1)$$

Posterior distributions



Results: Parameter estimation for SI $= 14$



	K	λ	δ
O	0.191	0.008	1.533
F	0.152	0.012	1.998
B	0.155	0.014	1.019

Modeling forest dynamics performance

Spatio-temporal marked point process

$$\Phi_M(t) = \{[X_i, M_i(t)] : i \in I_t\},\ t \in [0, T),\ T \in (0, \infty)$$

- **Points**:
  locations of trees (X_i)
- **Marks**:
  radius at breast height (M_i)
- **Dynamics**:
  immigration (birth), death,
  growth and interactions

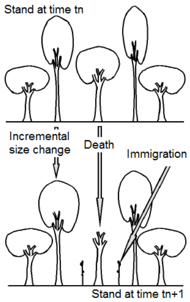
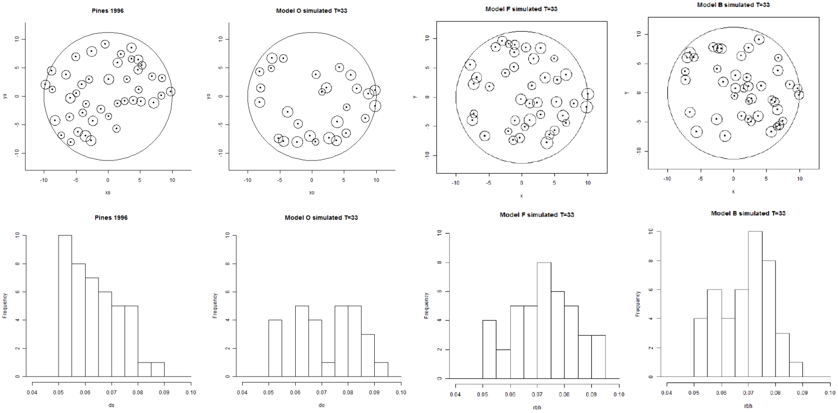


Image inspired by J.K. Vanclay (1994)

Data 1996 vs. model realizations



Summary and discussion

- Richards growth function was used to model the open growth of Swedish Scots pines
- Three model approaches were investigated
  - Frequentist fixed effects model
  - Frequentist mixed effects model
  - Two-level hierarchical Bayes model
- Bayesian model output easier to interpret and to use in model construction than mixed model
- Next steps:
  - Add lag parameter to growth model
  - Study non-constant variance in the Bayesian approach

References

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[3] G. Seber, C. Wild. *Nonlinear Regression*. Wiley series in probability ans mathematical statistics. John Wily & Sons, Inc., Hoboken, 2003.

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[5] The Swedish forest industries - Facts and figures 2010. Skogs Industrierna, Stockholm, 2011.

[6] J.K. Vanclay. *Modelling forest growth and yield: applications to mixed tropical forests*. CAB International, Wallingford, 1994.

Thank you for your attention