An introduction to Bayesian computation & evidence synthesis using Stan

mc-stan.org

About the speaker

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Wrote the StataStan interface

Interested in Bayesian latent variable models

robertgrantstats.co.uk

Bayesian computation

Computer-intensive methods
Simulation

Metropolis algorithm (40s)
Metropolis-Hastings algorithm (70s)
Gibbs sampler (80s)
Hamiltonian Monte Carlo (80s)

Bayesian software

M-H / Gibbs: BUGS, JAGS, JASP, SAS (proc mcmc), Stata (bayesmh)

Hamiltonian MC: Stan

Hamiltonian Monte Carlo

Speed (rotation-invariance + convergence + mixing)

Flexibility of priors

Stability to initial values

See Radford Neal's chapter in the *Handbook of MCMC*

Hamiltonian Monte Carlo

Tuning is tricky

One solution is the No U-Turn Sampler (NUTS)

Stan is a C++ library for NUTS (and variational inference, and L-BFGS)

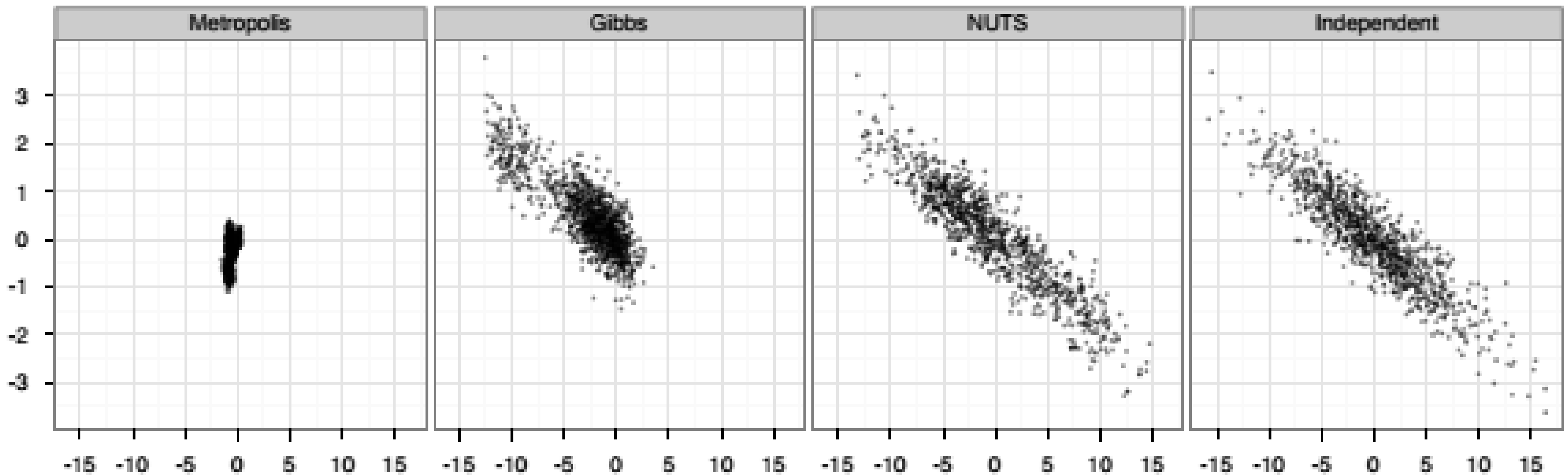


Figure 7: *Samples generated by random-walk Metropolis, Gibbs sampling, and NUTS. The plots compare 1,000 independent draws from a highly correlated 250-dimensional distribution (right) with 1,000,000 samples (thinned to 1,000 samples for display) generated by random-walk Metropolis (left), 1,000,000 samples (thinned to 1,000 samples for display) generated by Gibbs sampling (second from left), and 1,000 samples generated by NUTS (second from right). Only the first two dimensions are shown here.*

http://www.stat.columbia.edu/~gelman/research/published/nuts.pdf

Some Stan model code

```
data {
      int N;
      real y[N];
      real x[N];
}
parameters {
      real beta[2];
      real<lower=0> sigma;
}
model {
      real mu[N];
      beta ~ normal(0,50);
      sigma ~ normal(0,20);
      for(i in 1:N) {
            mu[i] <- beta[1] + beta[2]*x[i];
      }
      y ~ normal(mu,sigma);
}
```

rstan

```
stan( file = 'model.stan',
      data = list.of.data,
      chains = 4,
      iter = 10000,
      warmup = 2000,
      init = list.of.initial.values,
      seed = 1234 )
```

CmdStan

make "C:\model.exe"

model.exe sample data file="mydata.R"

stansummary.exe output.csv

StataStan

```
global cmdstandir "C:/cmdstan-2.9.0"

quietly count
global N=r(N)

stan y x1 x2 x3, modelfile("model.stan") ///
cmd("$cmdstandir") globals("N")
```

Some simulations

Collaboration with Furr, Carpenter, Rabe-Hesketh, Gelman
arxiv.org/pdf/1601.03443v1.pdf
rstan v StataStan v JAGS v Stata

More recently: rstan v rjags
robertgrantstats.co.uk/rstan_v_jags.R

Rasch model (item-response)

$$\Pr(y_{ip} = 1 | \theta_p, \delta_i) = \text{logit}^{-1}(\theta_p + \delta_i)$$

$$\theta_p \sim \mathrm{N}(0, \sigma^2)$$

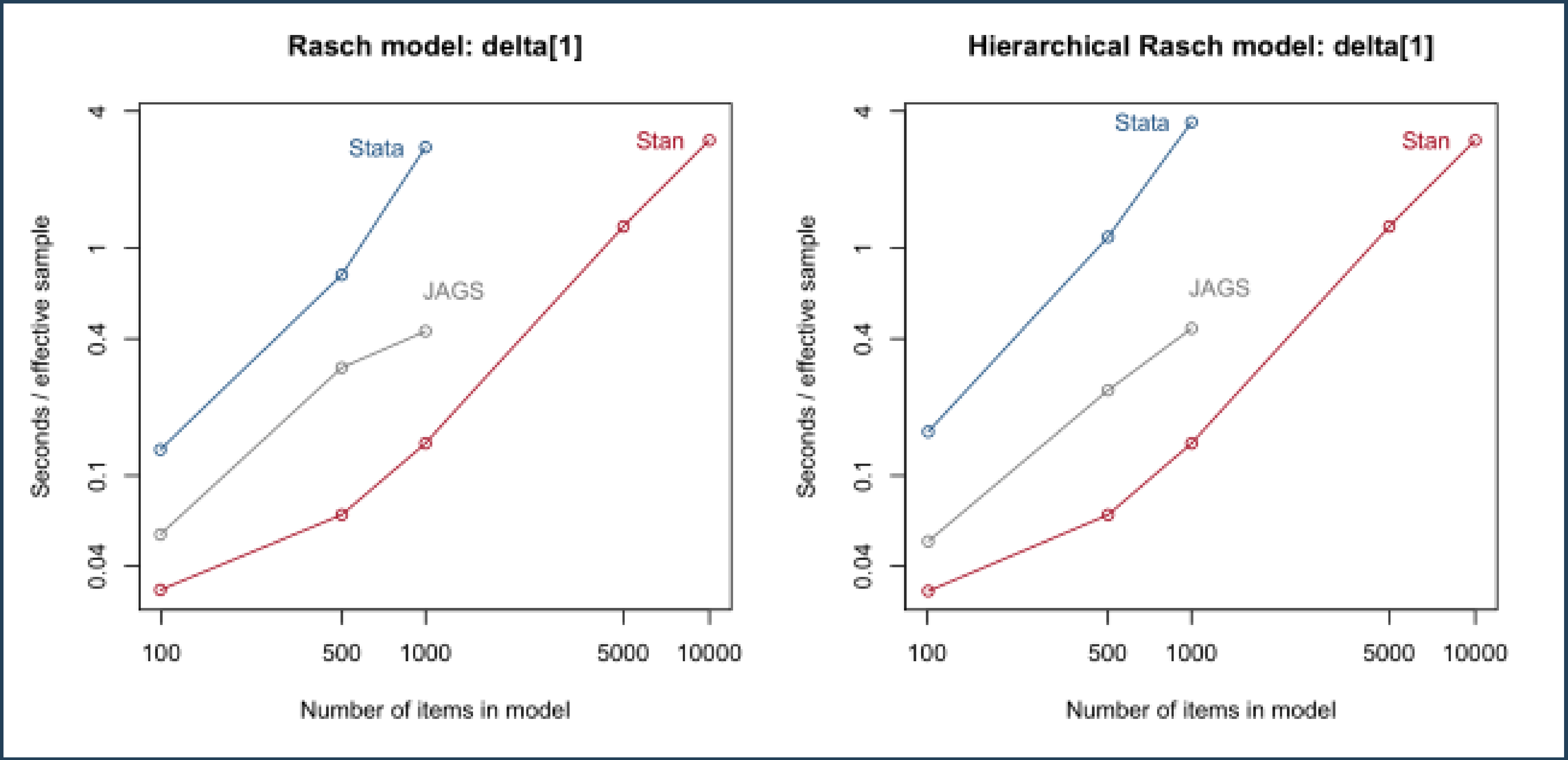
Hierarchical Rasch model (includes hyperpriors)

$$\Pr(y_{ip} = 1 | \mu, \theta_p, \delta_i) = \text{logit}^{-1}(\mu + \theta_p + \delta_i)$$

$$\theta_p \sim \mathrm{N}(0, \sigma^2)$$

$$\delta_i \sim \mathrm{N}(0, \tau^2)$$

StataStan vs Stata vs rjags



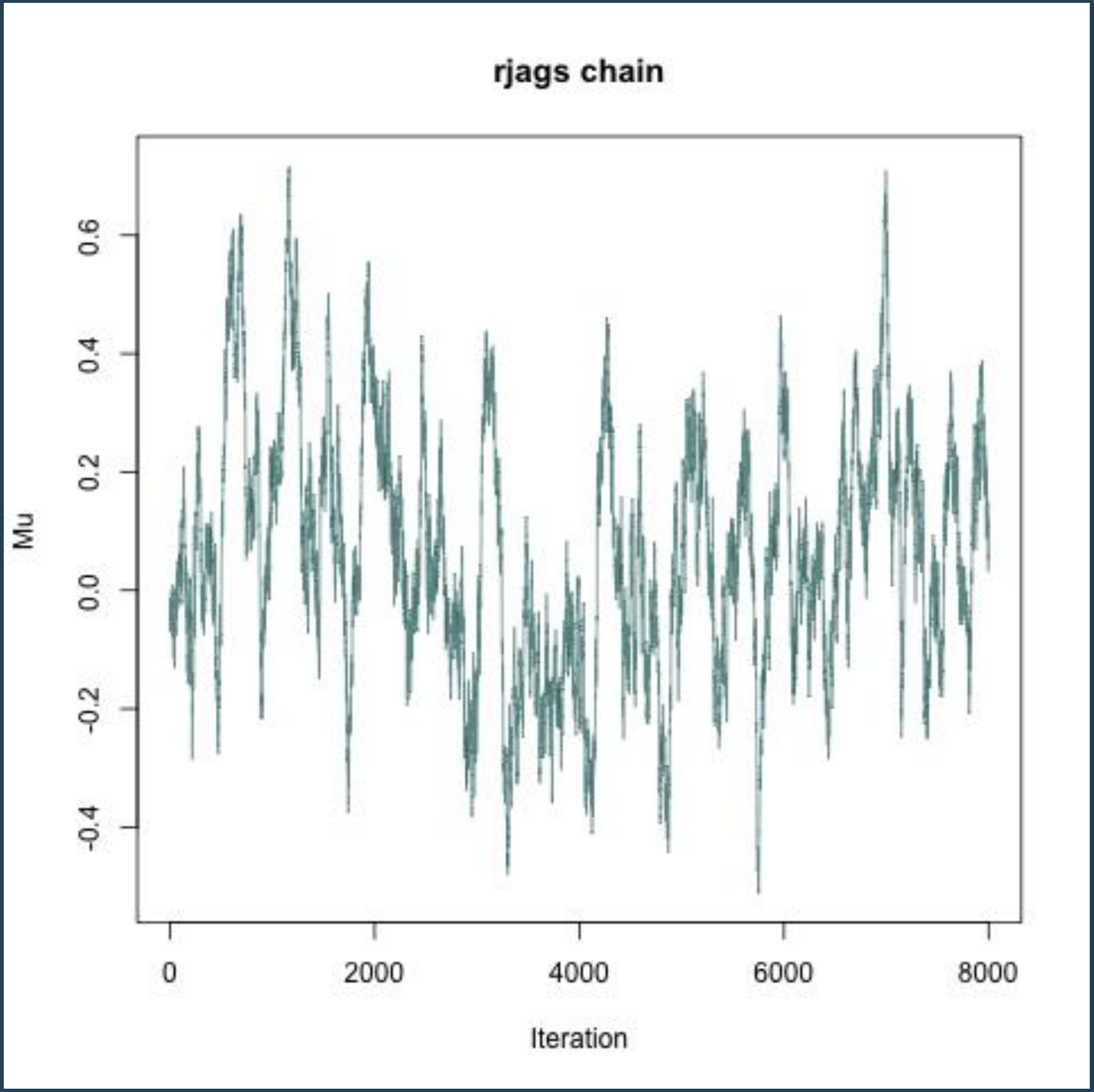
rstan vs rjags

Seconds:	Rasch	H-Rasch
rstan	180	210
rjags	558	1270

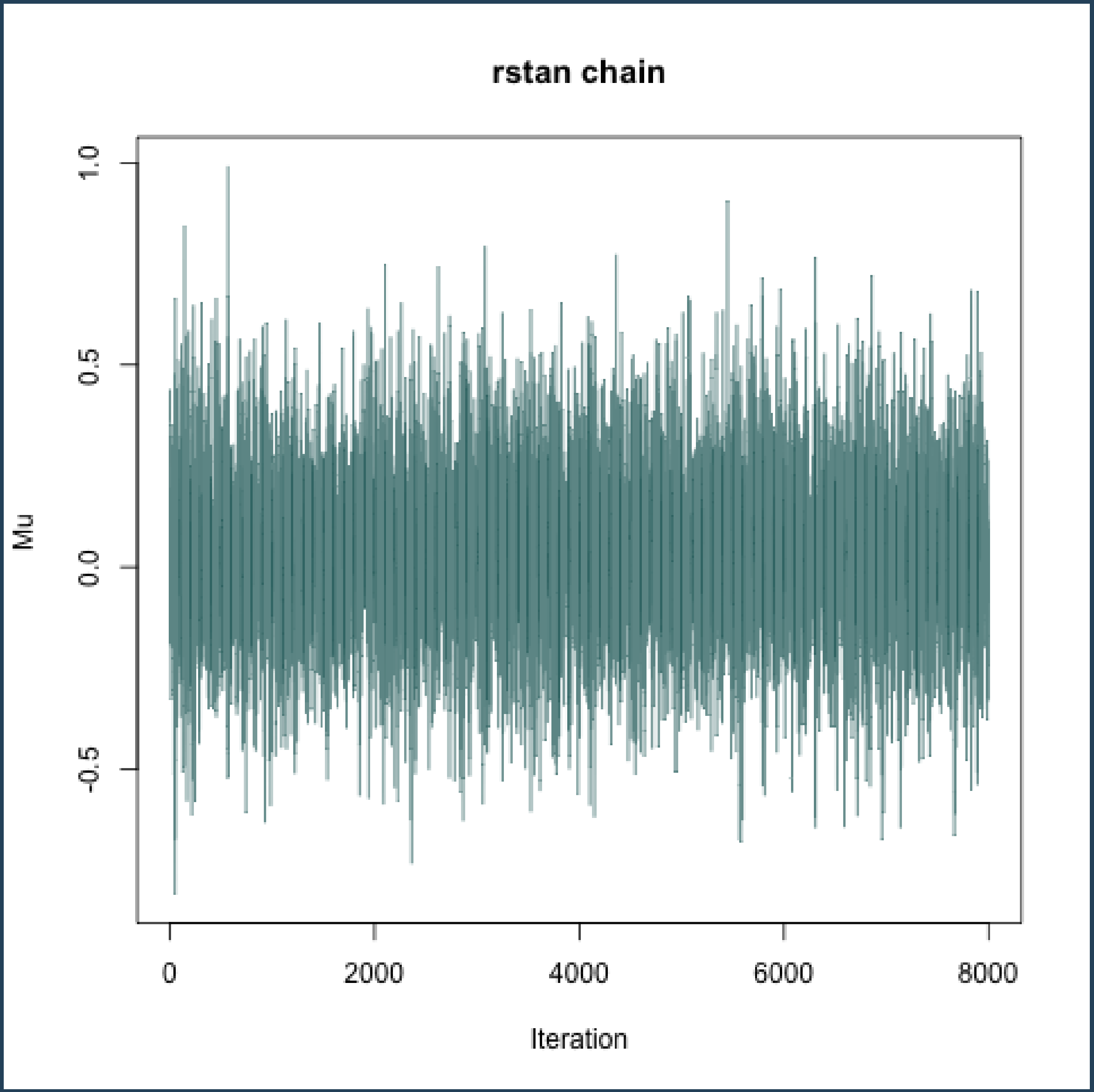
ESS (sigma):	Rasch	H-Rasch
rstan	22965	21572
rjags	7835	8098

ESS (theta1):	Rasch	H-Rasch
rstan	32000	32000
rjags	19119	19637

rstan vs rjags



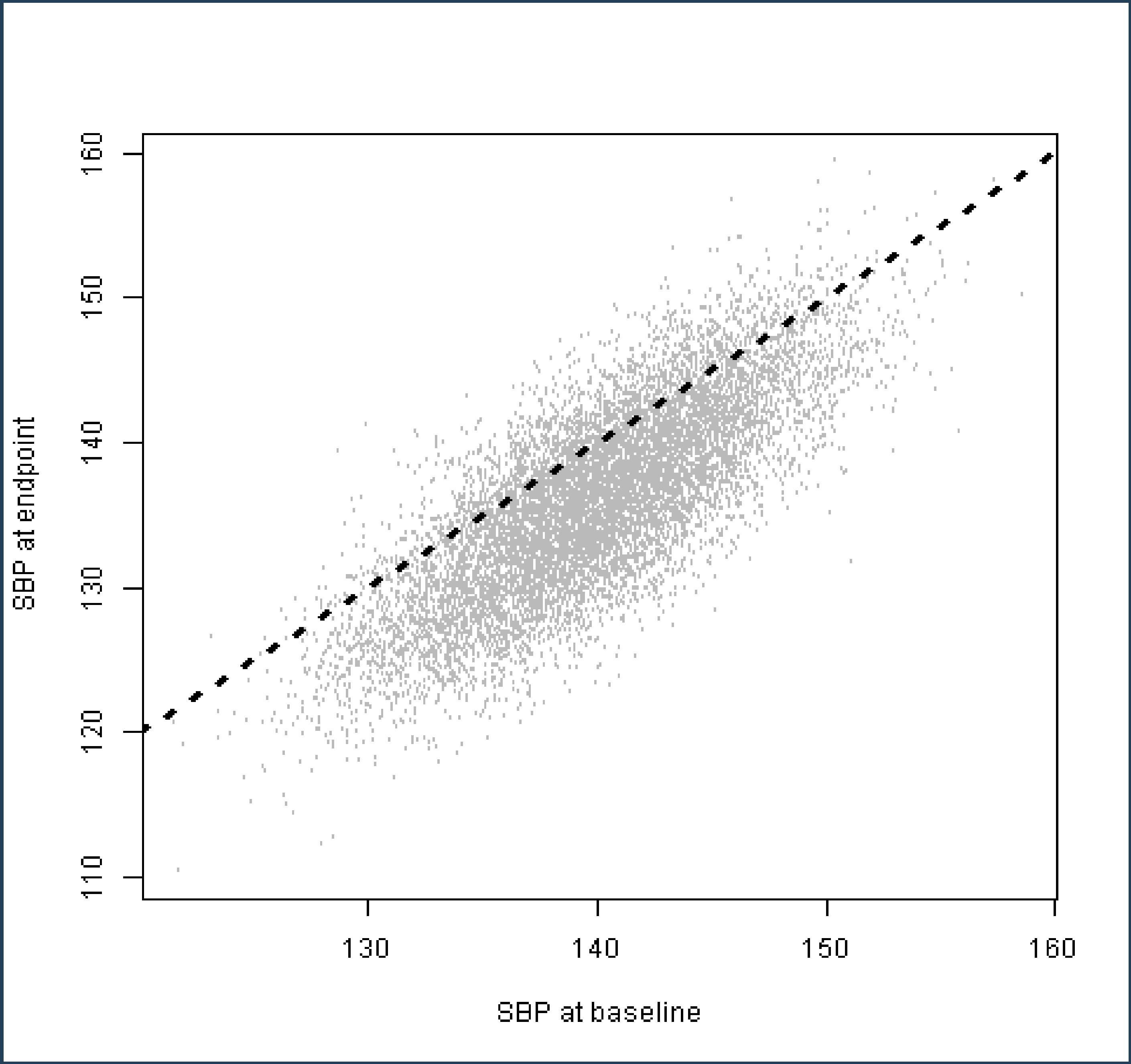
rstan vs rjags



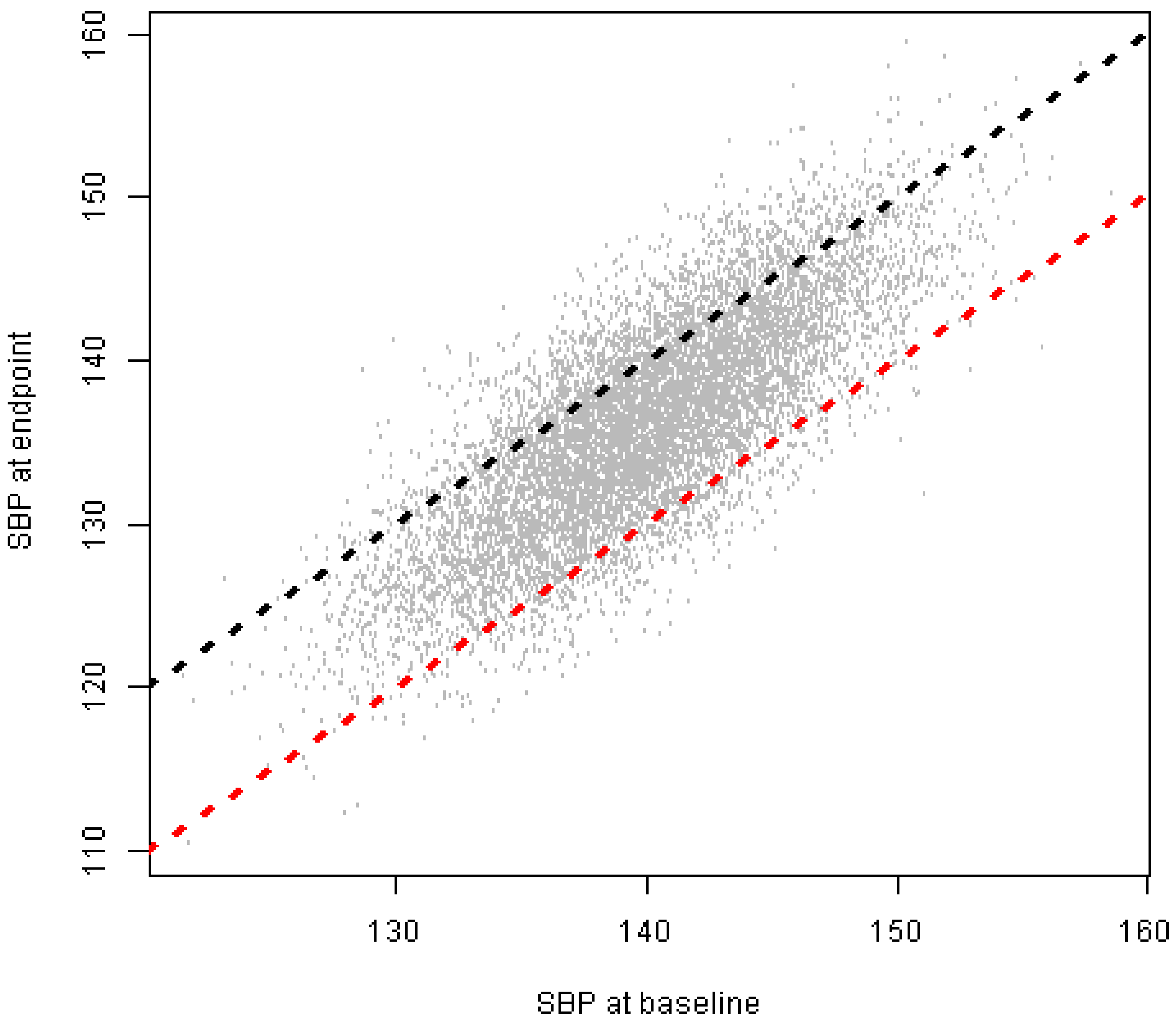
Evidence synthesis

Bayesian models can go beyond crude approximations

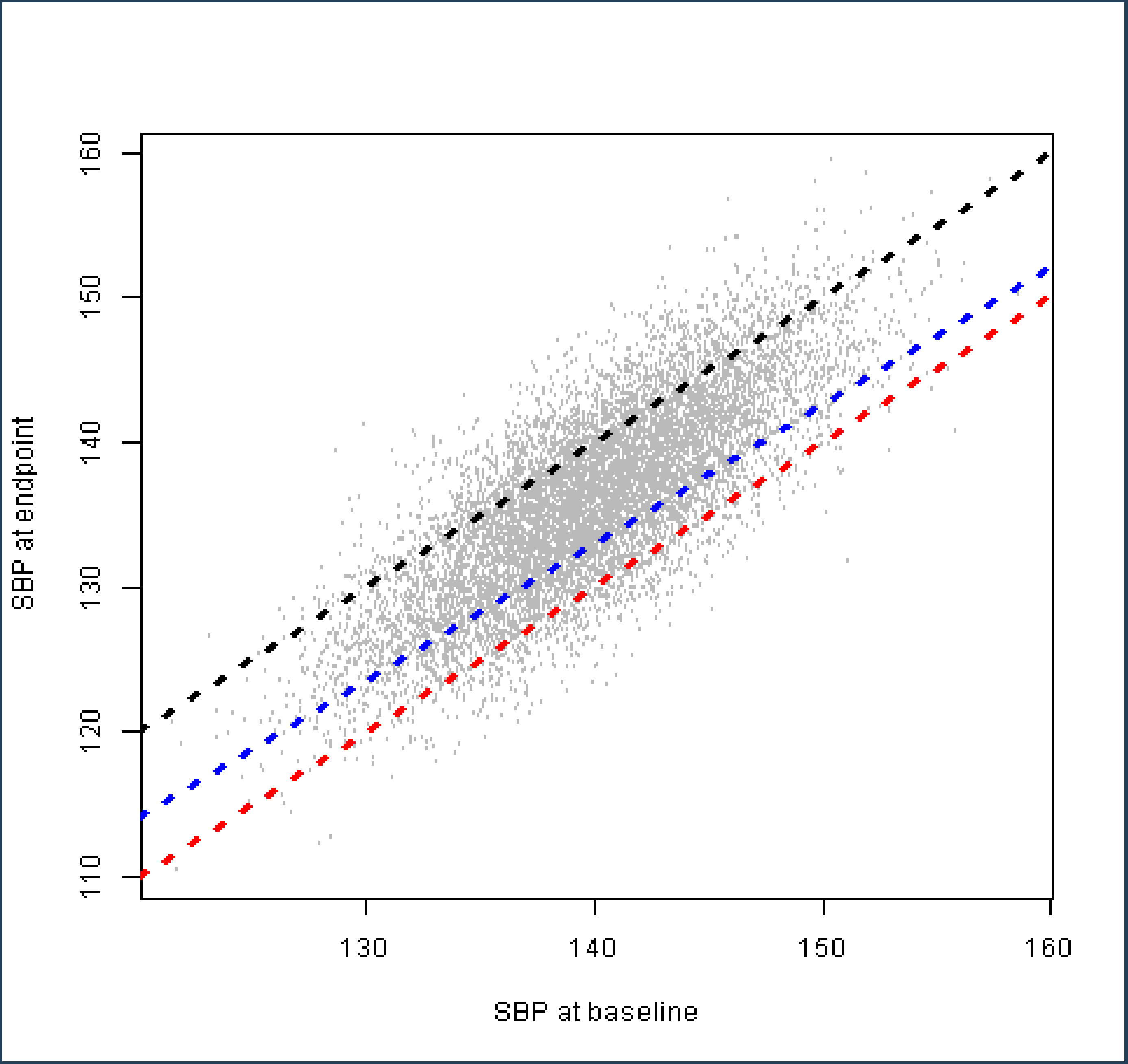
Different statistics
Different metrics
Different scales
Other uncertainty & bias



SBP at endpoint
SBP at baseline
110
120
130
140
150
160
130
140
150
160



SBP at endpoint
110
120
130
140
150
160
SBP at baseline
130
140
150
160



160
150
140
130
120
110
SBP at endpoint
130
140
150
160
SBP at baseline

Coarsened data

See Heitjan & Rubin 1990

Given proportion achieving a threshold at endpoint, and baseline statistics, we can work out a posterior conditional distribution for the endpoint means.

We may have to assume, model or simulate SDs, correlation...

Test case

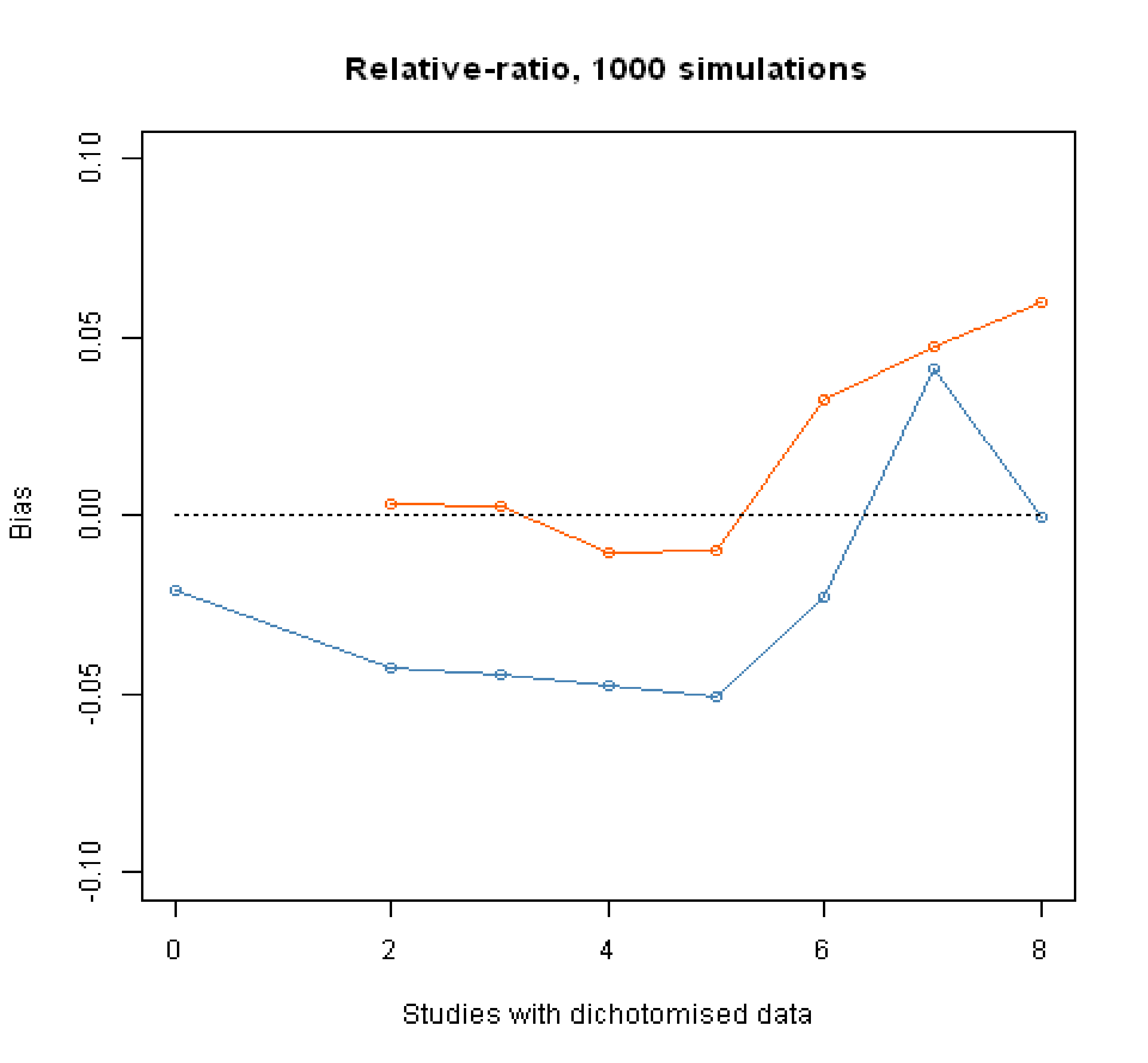
Cochrane review of tricyclic antidepressants in children (latest update: Mizraei et al 2013)

13 trials, sample size between 6 and 173

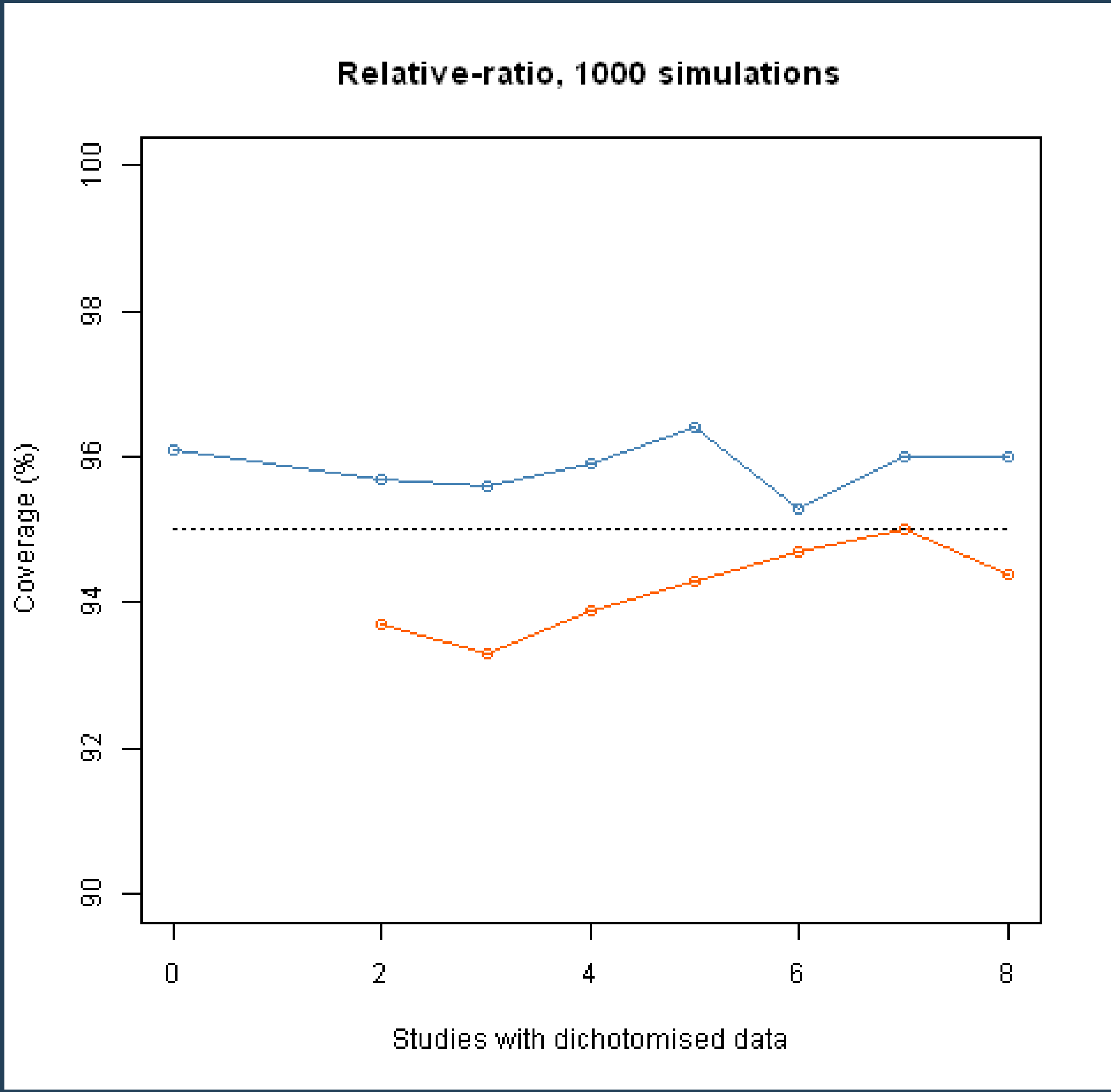
8 trials: mean differences & responders, one responder only

Mostly relative-ratio, but some ambiguity

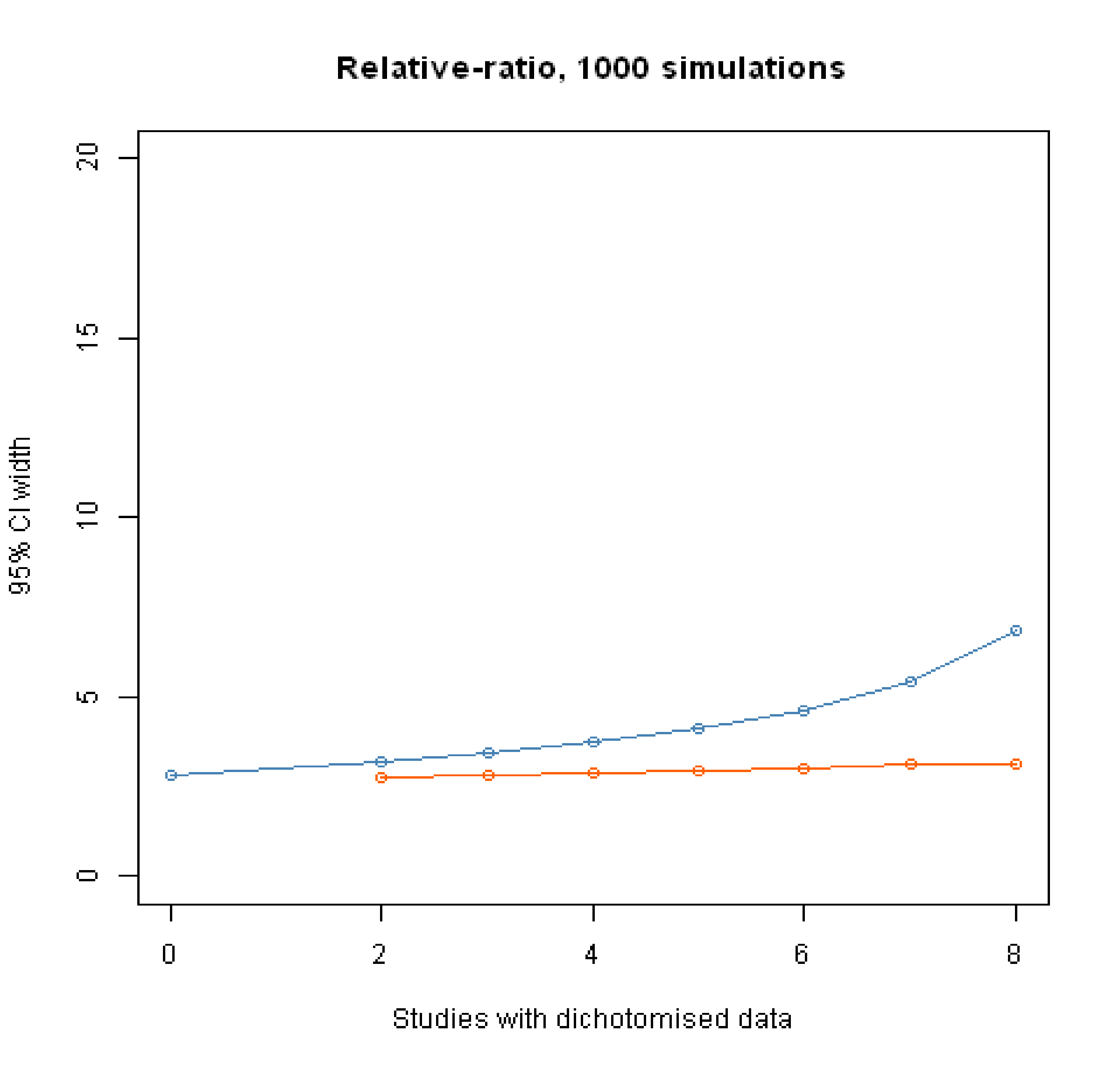
Simulation study



Simulation study



Simulation study



Cochrane review results

From Mizraei et al:
mean reduction (SMD) of 0.32
    CI 0.04 to 0.59
risk ratio for responding: 1.07
    CI 0.91 to 1.26
From the Bayesian model:
mean reduction (on CDRS scale) of 3.8 points
    CI 2.4 to 5.4

A more complex setting

Review of psycho-social benefits of exercise in osteoarthritis

Lots of differences among studies
Change from baseline vs endpoint
Duration of intervention
A structural equation model

Getting started

mc-stan.org

stan-users Google Group